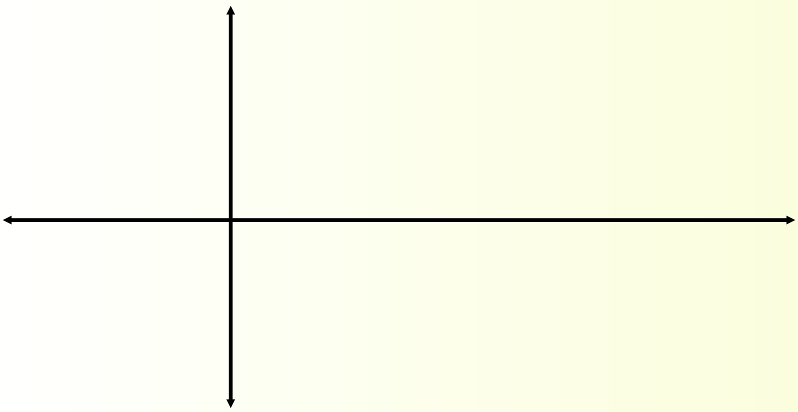
Alg. 2 Warm Up #10-4

1. Describe the transformations of y = sinx that give us y = - 4 sin(2x) + 1, then graph one cycle.

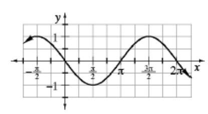
Label the line of oscillation.



Preview

HW Questions:

7-158. Susan knew how to shift $y = \sin x$ to get the graph at right, but she wondered if it would be possible to get the same graph by shifting $y = \cos x$.



a. Is it possible to write a cosine function for this graph? $y = \cos(x + \frac{\pi}{2})$

b. If you think it is possible, find an equation that does it. If you think it is impossible, explain why.

c. Adlai said, *"I can get that graph* without *shifting to the right or left*." What equation did he write? $y = -\sin x$

7-159. In the function $y = 4\sin(6x)$, how many cycles of sine are there from 0 to 2π? How long is each cycle (i.e., what is the period)? 6 $\frac{2\pi}{6} = \frac{\pi}{3}$

7-160. Write the equation of a cyclic function that has an amplitude of 7 and a period of 8π. Sketch its graph.

$y = 7\sin bx$

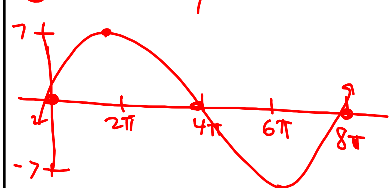
$y = 7\sin\frac{1}{4}x$

$Per = \frac{2\pi}{b}$

$\frac{8\pi}{1} = \frac{2\pi}{b}$

$\frac{8\pi(b)}{8\pi} = \frac{2\pi}{8\pi}$

$b = \frac{1}{4}$



7-161. Recall the strategies you developed for converting degrees to radians. How could you reverse that? Convert each of the following angle measures. Be sure to show all of your work.

use: $\frac{180°}{\pi}$ or $\frac{\pi}{180°}$

a. π radians to degrees

b. 3π radians to degrees

c. 30 degrees to radians

d. $\frac{\pi}{4}$ radians to degrees

e. 225 degrees to radians $\frac{225°}{1} \cdot \frac{\pi}{180°}$

$\frac{5\pi}{4}$

f. $\frac{3\pi}{2}$ radians to degrees

7-162. Find the exact value for each of the following trig expressions. For parts (g) and (h), assume that $0 \le \theta \le 2\pi$.

a. $\cos\left(\frac{3\pi}{4}\right) =$ ← x coordinate on unit circle.

b. $\tan\left(\frac{4\pi}{3}\right) =$

c. $\sin\left(\frac{11\pi}{6}\right) =$

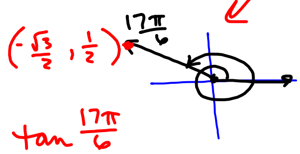
d. $\sin\left(\frac{3\pi}{4}\right) =$

e. $\tan\left(\frac{5\pi}{4}\right) =$

f. $\tan\left(\frac{17\pi}{6}\right) =$

g. $\tan(\theta) = 1$

h. $\tan(\theta) = -1$



Where is $\frac{\sin\theta}{\cos\theta} = -1$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\tan \frac{17\pi}{6}$

$= \tan \frac{5\pi}{6}$

$= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} \rightarrow -\frac{\sqrt{3}}{3}$

7-163. Solve this system of equations:

$$\begin{aligned} (1)\quad & 5x - 4y - 6z = -19 \\ (2)\quad & (-2x + 2y + z = 5)2 \\ (3)\quad & 3x - 6y - 5z = -16 \end{aligned}$$

2(2) + (1)

$$\begin{aligned} -4x + 4y + 2z &= 10 \\ 5x - 4y - 6z &= -19 \\ \hline x - 4z &= -9 \end{aligned}$$

3(2) + (3) →

$$\begin{aligned} -6x + 6y + 3z &= 15 \\ 3x - 6y - 5z &= -16 \\ \hline -3x - 2z &= -1 \end{aligned}$$

7-163. Solve this system of equations: $5x-4y-6z=-19$
$-2x+2y+z=5$
$3x-6y-5z=-16$

$\boxed{2^{nd}}$ Matrix

3×4 $\begin{bmatrix} 5 & -4 & -6 & -19 \\ -2 & 2 & 1 & 5 \\ 3 & -6 & -5 & -16 \end{bmatrix}$

$(-1, \frac{1}{2}, 2)$

$\boxed{2^{nd}}$ Matrix $\rightarrow$ Math rref

7-164. Use the Zero Product Property to solve each equation in parts (a) and (b) below.

a. $x(2x+1)(3x-5)=0$

b. $(x-3)(x-2)=12$

c. Write an equation and show how you can use the Zero Product Property to solve it.

$x^2-5x+6=12$
$\quad -12 \quad -12$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

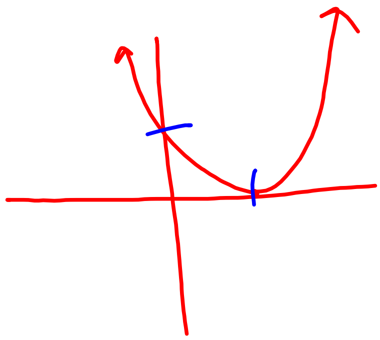
$x=6,-1$

7-165. Find a quadratic equation whose graph has each of the following characteristics:

a. No x-intercepts and a negative y-intercept.

b. One x-intercept and a positive y-intercept.

c. Two x-intercepts and a negative y-intercept.



x = yrs since 2005

7-166. A two-bedroom house in Seattle was worth \$400,000 in 2005. If it appreciates at a rate of 3.5% each year:

$x = 10$ plug it in

100% + 3.5%

103.5%

a. How much will it be worth in 2015?

b. When will it be worth \$800,000?

multiplier = 1.035

$y = 400{,}000(1.035)^x$

7-166. A two-bedroom house in Seattle was worth \$400,000 in 2005. If it appreciates at a rate of 3.5% each year:

a. How much will it be worth in 2015?

b. When will it be worth \$800,000?

c. In Jacksonville, houses are depreciating at 2% per year. If a house is worth \$200,000 now, how much value will it have lost in 10 years?

$100\% - 2\% = 98\%$

$y = 200{,}000(0.98)^x$

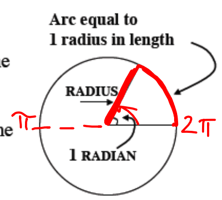
METHODS AND MEANINGS

p. 334

MATH NOTES

Radians

A **radian** is defined as an angular measure such that an arc length of one radius on a circle of radius one produces an angle with measure one radian. It can also be thought of as the ratio of an arc length to the radius of the corresponding circle.



The circumference of any complete circle is $2\pi r$ units, so the corresponding radian measure is $\frac{2\pi r}{r} = 2\pi$. Thus, there are 2π radians in a complete circle.

$\pi \approx 3.14$ radian

$2\pi \approx 6.28$ radians

1 radian $\approx 57°$

$\pi = 180°$

$2\pi = 360°$

Convert

$\theta \cdot \frac{180°}{\pi \text{ rad.}}$ or $\theta \cdot \frac{\pi \text{ rad.}}{180°}$

Math Notes — Methods and Meanings

p. 338

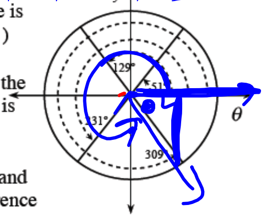
Reference Angle

$\theta' =$ Reference Angle

acute, positive

always next to x-axis

For every angle of rotation, there is an angle in the first quadrant ($0 \le \theta \le 90°$) whose cosine and sine have the same absolute values as the cosine and sine of the original angle. This first-quadrant angle is called the **reference angle**.



For example, the angles 51°, 129°, 231°, and 309° (pictured at right) all share the reference angle of 51°.

Quad I $\theta \rightarrow \theta'$

Quad II $180 - \theta = \theta'$

Quad III $\theta - 180° = \theta'$

Quad IV $360 - \theta = \theta'$

Math Notes — Methods and Meanings

p. 343

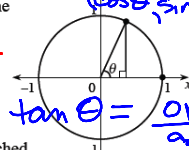
Sine, Cosine, and Tangent

radius = 1

(x, y)

$(\cos\theta, \sin\theta)$

For any real number θ, the **sine of θ**, denoted $\sin\theta$, is the y-coordinate of the point on the unit circle reached by a rotation of θ radians from **standard position** (counter-clockwise starting from the positive x-axis).



$\tan\theta = \frac{opp}{adj}$

The **cosine of θ**, denoted $\cos\theta$, is the x-coordinate of the point on the unit circle reached by a rotation of θ radians from standard position.

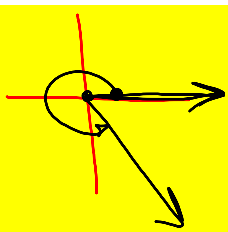
$\tan\theta = \frac{y}{x}$

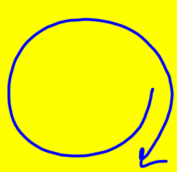
The **tangent of θ**, denoted by $\tan\theta$, is the slope of the terminal ray of an angle (the radius) formed by a rotation of θ radians in standard position.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

The $\boxed{\textbf{Pythagorean Identity}, \sin^2\theta + \cos^2\theta = 1}$ describes the relationship between the side lengths of a right triangle formed in a unit circle with the radius as the hypotenuse.

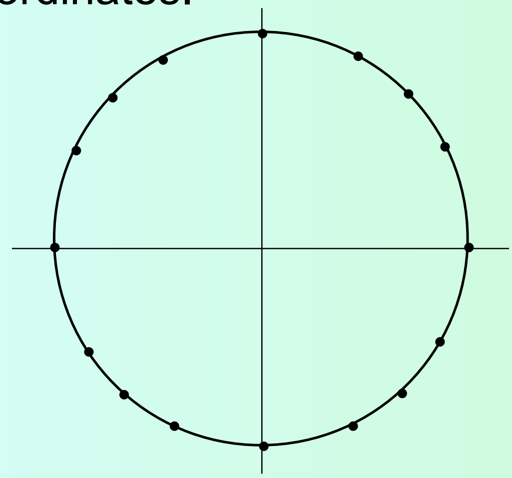
$(\sin\theta)^2 + (\cos\theta)^2 = 1$





Unit circle again!

Complete the pop quiz Special Triangles and Unit Circle with exact radian measures and coordinates.



Compare unit circles in your team.

Carefully check accuracy!

Resolve any disagreements.

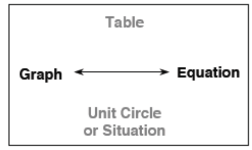
CP's: 7- #152 ---> 155 (over 2 days)

P. 358

7.2.4 What are the connections?

Graph ↔ Equation

In the past few lessons, you have been developing the understanding necessary to graph a cyclic equation without making a table and to write an equation from a cyclic graph. In today's lesson, you will strengthen your understanding of the connections between a cyclic equation and its graph. By the end of this lesson, you will be able to answer the following questions:



Does it matter if we use sine or cosine?

What do we need to know to make a complete graph or write an equation?

together:

7-152. What do you need to know about the sine or cosine functions to graph them or write their equations? Talk with your team and write a list of all of the attributes of a sine or cosine function that you need to know to write an equation and graph it.

Shape of the parent graph.
Amplitude
Period.
Line of Oscillation
Any vertical or horizontal translations

7-153. CREATE-A-CURVE

Split your team into pairs. With your partner, you will create your own sine or cosine function, write its equation, and draw its graph. Be sure to keep your equation and graph a secret! Start by choosing whether you will work with a sine or a cosine function.

a. Half the distance from the highest point to the lowest point is called the **amplitude**. You can also think of amplitude as the vertical stretch. What is the amplitude of your function?

b. How far to the left or right of the y-axis will your graph begin? In other words, what will be the **horizontal shift** of your function?

c. How much above or below the x-axis will the center of your graph be? In other words, what will be the **midline** of your function? Line of Oscillation

d. What will the **period** of your function be?

e. What will the **orientation** of your graph in relation to $y = \sin x$ or $y = \cos x$ be? Is it the same or is it flipped?

f. Now that you have decided on all of the attributes for your function, write its equation.

Your individual paper should have answers a-f, your created equation and its graph.

7- #154 Activity:

Copy your equation on a clean sheet of paper. Write your team number at the top. Pass it to the next team.

Carefully sketch the graph for the equation you received from the other team.

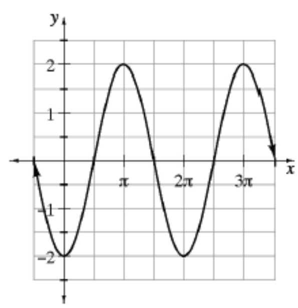
The original team will check your graph for accuracy.

Give kind and detailed feedback!

Check amplitude, period, line of oscillation, axes labeled with correct units, shifts, reflection, smooth curve, arrows for continues...

7-155. When you look at a graph and prepare to write an equation for it, do you think it matters if you choose sine or cosine? Which do you think will work best?

With your team, find *at least four* different equations for the graph at right. Be prepared to share your equations with the class.



a. Did it matter if you choose sine or cosine?

b. Which of your equations do you prefer? Why?

Practice: Solve by completing the square. (Simplify, exact, no decimals)

$3x^2 + 15x - 2 = 0$

Solve, check for extraneous solutions:

$\sqrt{3x - 3} - \sqrt{2x} = 1$

Practice: Solve by completing the square. (Simplify, exact, no decimals)

$3x^2 + 15x - 2 = 0$

$x = -\frac{5}{2} \pm \frac{\sqrt{249}}{6}$

$3(x^2 + 5x + \frac{25}{4}) = 2 + \frac{75}{4}$

Solve, check for extraneous solutions:

$\sqrt{3x - 3} - \sqrt{2x} = 1$

$x = 8 + 4\sqrt{3}$

$\sqrt{3x-3} = 1 + \sqrt{2x}$

then square both sides.

HW: 7-

#168 ---> 175

Group Quiz: Friday

(Grapher and notes ok. Topics:
Graphing a transformed sine graph
Solving equations with radicals
Solve by completing the square
Special Triangles)

Unit Circle: Monday
Test 7: Tuesday